Does "Proof" imply successful demonstration of truth? How can we differentiate between uses of the word by scientists, mathematicians, philosophers, and the public?

On The Nature of Proof

The concept of "proof" is contentious. Scientists, mathematicians, philosophers and the layperson may use the word in strikingly different ways, leading to arguments about what constitutes a proof, and how much can ultimately be known through the methods of proof. Providing care is taken, it is possible to reconcile these different uses of the word, but even after this reconciliation proof does not necessarily imply the successful demonstration of truth.

Truth

Proof is intrinsically linked with the idea of truth, and there are a multitude of philosophical theories regarding the nature of truth. To determine if proof does imply a successful demonstration of truth, we must first establish which (if any) of these definitions is most appropriate and if "truth", like proof, differs in meaning when used by different groups or individuals.

The correspondence theory of truth suggests that a proposition ("truthbearer") is true if and only if it responds to some real-world fact[1]. Coherence theory, on the other hand, suggests that "the truth of any (true) proposition consists in its coherence with some specified set of propositions"[2].

Leibniz also separated what he termed "necessary" from "contingent" truths. Necessary truths are those "that can be demonstrated through an analysis of terms"[3] – such a statement is true because

[1] David, Marian, "The Correspondence Theory of Truth", *The Stanford Encyclopedia of Philosophy (Fall 2005 Edition)*, Edward N. Zalta (ed.), Retrieved June 2007 from http://plato.stanford.edu/archives/fall2005/entries/truth-correspondence

[2] Young, James O., "The Coherence Theory of Truth", *The Stanford Encyclopedia of Philosophy (Summer 2001 Edition)*, Edward N. Zalta (ed.), Retrieved June 2007 from http://plato.stanford.edu/archives/sum2001/entries/truth-coherence

of the meaning of the words. “A bachelor is not married” is necessarily true because of how we define “bachelor” and “married”. An unmarried bachelor would be a contradiction, and “necessary truths depend upon the principle of contradiction.” [4]

Contingent truth, however, “cannot be reduced to the principle of contradiction”[5]. “This apple is red” would be viewed as a coherent truth. “In no coherent manner could God have created a five-sided square; but had he altered every object in the cosmos to account for it, he may have made this apple yellow.”[6]

Some statements are “true” under one or more of these definitions, but with others it is less obvious.

> *“If I say to you that the bird singing on the branch there is a warbler, how would you discover the truth of my statement? The truth of a statement that names something can be confirmed in a book of names, or by asking a speaker of the language in which the name is uttered. If I tell you that my servant can swim, the truth of the statement can be discovered by throwing him in the river; if I tell you that in my country the sun sets at midnight on midsummer day you will need to travel there at the right season to confirm it”*[7]

Each of these statements above may be “true” in some way, but each requires its own type of “proof”. We know the bird is a warbler, once we know that the word “warbler” describes this type of bird. We know the servant can swim because we can demonstrate it. We also know that the sun sets at midnight because we can observe it.

[3] Leibniz, G., Ariew, R., & Garber, D. (1989). *Philosophical Essays*. Indianapolis: Hackett Pub. Co., p. 28
[4] Leibniz, G., et al (1989)., p. 28
[5] Leibniz, G., et al (1989)., p. 28
[6] Ratcliff, Jason Stuart (n.d.) *Leibniz: Types of Truth*. Retrieved June 2007 from http://www.angelhaunt.net/leibniz/truth.html
[7] Shapiro, S. (1998). *Proof and Truth: Through Thick and Thin*. The Journal of Philosophy , 95 (10), p. 494

Our bird is a warbler, because how we define "warbler" relates to the specific characteristics of the bird, and thus this statement is *necessarily* true (using Leibniz's definitions), while also *corresponding* to a "fact" in the world. Also, if we define a set of propositions such that a bird is a warbler if and only if it has certain characteristics, we can demonstrate the *coherence* of our statement ("this bird is a warbler") with those propositions.

Using Leibniz's definitions, "my servant can swim" and "the sun sets at midnight" would both be *contingent* truths, as it would be possible for these to be false without being contradictory. These would also be *coherently* true if, indeed, my servant can swim, and the sun does actually set at midnight.

Each of these examples, however, relies on some form of axiom or simpler truth – we need to trust our reference book to know the true name of the bird, and we must trust in our own senses to confirm the truth of the other statements. A *coherent* truth can only be demonstrated by relying on the necessary truth of the initial propositions or axioms. Similarly, to know the truth of a "corresponding" truth requires knowledge of the fact with which it corresponds. In other words, whichever definition we choose, to "prove" these statements requires a foundation on some other "known truth". As such, it seems Leibniz's distinction between necessary and contingent truths is as good a method as any to differentiate between axiomatic truths and those which we generate from them.

Everyday Concepts of "Proof"

In everyday language, "proof" may suggest the *process* of "marshalling of premises in support of some claim, *in the context of trying to convince someone of something*"[8], or it might refer to the resultant *product*. We might hear claims of a "proof of a Government conspiracy", or "proof that God exists" but, in many cases, it should become clear that "proof" in this sense is unlikely to be a

[8] Aspenson, S., (1998). *The Philosopher's Tool Kit*. Armonk: M.E. Sharpe, pp.79-79.

successful demonstration of truth (necessary or contingent). The lack of words such as “misprove” [9] in our language suggests that proof does not necessarily mean “guarantee of certain truth”[10], and in the common usage, a “proof” is often no more than an argument.

In two seemingly typical examples of everyday proof, we certainly see convincing argument:

A. “You shaved your head? Prove it!”
B. (removes hat to expose shaved head)[11]

Or

A. Where's the butter?
B. In the fridge.
A. Are you sure?
B. (Proof:) I just *saw* it[12]

In both of these examples there is no certainty – neither of these propositions is necessarily true. It is possible that I could have hallucinated your bald head and you may have lied (or been mistaken) about seeing the butter. There is no contradiction if your head was not shaved, or the butter was not in the fridge. In general usage, however, the evidence given seemingly constitutes sufficient “proof” to know the truth of the propositions.

In both of the above examples, the “proof” was supplied in order to convince someone of something. While a philosopher or mathematician may not be interested in convincing anyone, the everyday use of the word certainly seems to include this requirement.

Perhaps the everyday use of the word “proof” might better be termed “evidence” in support of an “argument”, or (more specifically) “evidence that is *sufficient* to establish knowledge of a conclusion”[13]. In the examples above, seeing your bald head, or trusting that you did, in fact, just see the butter would be sufficient evidence for me to know that your head is bald and the butter is in the fridge. In both cases, however, it is clear that the “proof” does not necessarily constitute a successful demonstration of truth.

[9] Aspenson, S., (1998), p.79
[10] Lakatos, I., (1976). *Proofs and Refutations*. Cambridge: Cambridge University Press, p.14
[11] Chappell, R. (June 17, 2005) “Evidence, Knowledge, and Proof”. Retrieved June 12, 2007 from http://pixnaps.blogspot.com/2005/06/evidence-knowledge-and-proof.html
[12] Aspenson, S., (1998), p.79
[13] Chappell, R. (June 17, 2005)

Scientific "Proof"

The scientific notion of proof is not dissimilar from that in general use. While scientists (or, perhaps, the media which portrays scientific research to the public) may occasionally be casual in their use of the word "proof", when questioned about the certainty of their results (and thus, whether their work actually constitutes "proof"), they might reply: "Scientists don't talk about 'proof', period. We leave that to the mathematicians... Change the word 'proof' to 'evidence', and it makes more sense."[14]

Scientists seem to have little problem admitting that they do not, in fact, prove anything. Karl Popper, whose ideas defined the modern scientific method, suggested that "scientific experimentation [is not] carried out with a view to verifying or finally establishing the truth of theories; ... we can never finally prove our scientific theories, we can merely (provisionally) confirm or (conclusively) refute them"[15]

It should be clear that any use of the word "proof" in science is either accidental, or a deliberate attempt to promote "pseudo-science" or non-science as legitimate. Scientific theories may *be* contingently true, but it is not the goal of science to provide "proof". While science may have higher standards of evidence than the general population, scientists (like the general population) seek "evidence that is sufficient to establish knowledge of a conclusion"[16], not certain truth.

Mathematical or Philosophical Proof

Despite dealing with very different areas of knowledge, mathematicians and philosophers both use similar definitions of proof. Mathematical or philosophical proof consists of logical arguments which

[14] Myers, P.Z. (2005, June 16) *"Volokh's question"* Retrieved June 12, 2007 from http://pharyngula.org/index/weblog/comments/volokhs_question/
[15] Thornton, Stephen, "Karl Popper", *The Stanford Encyclopedia of Philosophy (Winter 2006 Edition)*, Edward N. Zalta (ed.), Retrieved June 12, 2007 http://plato.stanford.edu/archives/win2006/entries/popper/
[16] Chappell, R. (June 17, 2005)

establish the contingent truth of a final statement, given the truth of certain axioms or assumptions.[17,18,19,20]

Given the possibility of false axioms or invalid logic, however, the certainty of the conclusion may be in doubt. To successfully demonstrate truth, a mathematical or philosophical proof must contain:

1. *True* axioms and assumptions (true “evidence”), and
2. *Valid* reasoning based only upon these axioms (valid “argument”), thus resulting in
3. A *true* conclusion (established “truth”)

While it seems mathematicians and philosophers may have a more definitive version of proof, the ability to know if a proof could be called a “successful proof” (one which successfully demonstrates truth) is a more difficult matter. It may be impossible to differentiate between a “successful” or “false” proof, unless the truth of the axioms and validity of the argument can be known a priori, or only with reference to known “successful” proofs. Or, using Leibniz’s definitions, only if the axioms are necessarily true can we be sure the conclusion is also true.

Conclusion

After reconciling the different uses of the word proof we can see that neither definition suggests that proof is a successful demonstration of truth. In everyday and scientific usage (with varying requirements for “evidence”) “evidence that is *sufficient* to establish knowledge of a conclusion”[21] may not result in truth. Similarly, the definition used by mathematicians and philosophers (“logical arguments which establish the contingent truth of a final statement, given the truth of certain axioms or assumptions”) gives no absolute certainty.

---

[17] Definition from: http://en.wikipedia.org/wiki/Proof_(math) – “In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true.”

[18] Definition from: http://wordnet.princeton.edu/perl/webwn?s=proof – “a formal series of statements showing that if one thing is true something else necessarily follows from it”

[19] Definition from: http://www.lhup.edu/~dsimanek/glossary.htm - “an argument from premise to conclusion using strictly logical principles”

[20] Definition from: http://ddi.cs.uni-potsdam.de/Lehre/TuringLectures/MathNotions.htm - “A proof is a sequence of statements (made up of axioms, assumptions and arguments) leading to the establishment of the truth of one final statement.”

[21] Chappell, R. (June 17, 2005)

In the everyday usage, proof is simply not conclusive. Even with higher standards of evidence required by scientists, many previously reputable scientific theories have since been refuted, suggesting that the previous “proof” was insufficient. Philosophically and mathematically, proofs rely on axioms which must be accepted as true, but cannot be demonstrated to be so. Therefore, even these proofs do not necessarily demonstrate certain truth – the result of a philosophical proof can only be as “true” as the propositions and reasoning on which it is based.

Bibliography

Aspenson, S. (1998). *The Philosopher's Tool Kit.* Armonk: M.E. Sharpe.

Chappell, R. (2005, June 17). *Evidence, Knowledge, and Proof.* Retrieved June 12, 2007, from Philosophy, et cetera: http://pixnaps.blogspot.com/2005/06/evidence-knowledge-and-proof.html

David, M. (2005, Fall). *The Correspondence Theory of Truth.* (E. N. Zalta, Ed.) Retrieved June 2007, from The Stanford Encyclopedia of Philosophy: http://plato.stanford.edu/archives/fall2005/entries/truth-correspondence

Lakatos, I. (1976). *Proofs and Refutations.* Cambridge: Cambridge University Press.

Leibniz, G., Ariew, R., & Garber, D. (1989). *Philosophical Essays.* Indianapolis: Hackett Pub. Co.

Myers, P. (2005, June 16). *Volokh's question.* Retrieved June 12, 2007, from Pharyngula: http://pharyngula.org/index/weblog/comments/volokhs_question/

Ratcliff, J. S. (n.d.). *Leibniz: Types of Truth.* Retrieved June 2007, from The Angelhaunt Library: http://www.angelhaunt.net/leibniz/truth.html

Shapiro, S. (1998). Proof and Truth: Through Thick and Thin. *The Journal of Philosophy , 95* (10), 493-512.

Thornton, S. (2006, Winter). *Karl Popper.* (E. N. Zalta, Ed.) Retrieved June 12, 2007, from The Stanford Encyclopedia of Philosophy: http://plato.stanford.edu/archives/win2006/entries/popper/

Young, J. O. (2001, Summer). *The Coherence Theory of Truth.* (E. N. Zalta, Ed.) Retrieved from The Stanford Encyclopedia of Philosophy: http://plato.stanford.edu/archives/sum2001/entries/truth-coherence